Synopsis

Ph.D. Mathematics

Operation Approach on αg-Open Sets and Neutrosophic Soft Open Sets

Submitted By

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INTRODUCTION

Topology is a fascinating branch of mathematics that explores the properties of space under continuous transformations. It deals with the properties of space and the relationships between objects within the space. Two objects are topologically equivalent if one object can be transformed into the other by twisting and stretching, but not tearing, cutting, or gluing. The flexibility of topology to capture the qualitative characteristics of geometric objects while ignoring their precise shape or size is a component that makes it so appealing to researchers. This versatility allows for a wide range of applications in various fields.

A key concept in topology is that of a topological space. Topological spaces can be classified based on their properties, and different types of spaces have distinct topological characteristics. For example, a metric space is a type of topological space that has a notion of distance defined on it, while a manifold is a space that locally resembles Euclidean space. A topological space is a set together with a collection of subsets called open sets that satisfy certain axioms. These axioms capture the intuitive notion of continuity and allow for the study of various properties of spaces, such as continuity, compactness, connectedness, and separability.

The open sets are the building blocks of the topology. Open sets are subsets of a topological space that satisfy the axioms of the topology. They are used to define the concept of neighborhoods and continuity of functions. Closed sets that are complement of open sets, are subsets of a topological space that include all their limit points. Closed sets are often used in conjunction with open sets to describe the properties of a space. They help to describe the concept of limits and completeness.

Eventhough concept of open sets and closed sets are fundamental concepts in topology, there are cases where a generalization of closed sets become necessary to capture additional properties or characteristics. Levine (1970) introduced gener-

alized closed sets, a novel concept which has a profound impact on topology and explored their properties and applications. His contributions have enhanced the understanding of closure properties in topology and extended the scope of analysis beyond traditional boundaries. His work opened up new avenues for research and has since influenced subsequent developments in the field. Levine's research extends beyond theoretical developments and encompasses practical applications of generalized closed sets in diverse mathematical domains. By extending the concept of closed sets to these contexts, he has contributed to the advancement of related theories and methodologies, broadening their scope and applicability. Various generalized open and closed sets have been established and their properties and structures have been investigated intensively in the past years.

The concept of closed sets was generalized by Bhattacharyya and Lahiri (1987) to semi-generalized closed sets. This notion has been studied extensively in the past few decades by many topologists. Arya and Nour (1990) defined generalized semi closed sets. Generalized α-closed set was introduced and discussed by Maki et al. (1993). Maki et al. (1994) introduced α-generalized closed sets.

Kasahara (1979) defined and investigated the concept of operations on ς. That is the function $\alpha : \varsigma \to P(\mathfrak{L})$ such that $U \subset U_\alpha$ for each $U \in \varsigma$, where U_α denotes the value $\alpha(U)$ of α at U and $P(\mathfrak{L})$ the power set of $\mathfrak{L}$. He proposed the idea of an operation approach to topological spaces and defined α-closed graphs of functions by genera1izing the idea of almost-strong1y-closed, strong1y-closed and closed graph of a function, α-continuous by genera1izing the idea of continuity, weak continuity and almost-continuity. He also defined regular operation on ς and unified several characterizations of compactness by using α-compactness. Janković (1983) investigated the mappings with α-closed graphs and strongly-closed graphs. He unified 1ocally closed, 1ocally ϑ-closed, 1ocally δ-closed using 1ocal1y α-closed maps. He proved that the class of α-closed functions is contained in the class of a1most α-closed functions. Also, he proved if the domain of an almost α-closed function is a regu1ar space, then the function is 1ocally α-closed.

Ogata (1991) renamed the operation α as γ and introduced γ-open sets by defining mapping on the collection of open sets in a topological space. In the realm of topology, the study of operations on topological spaces plays a crucial role in understanding the relationships and transformations that can occur within

the spaces. He investigated how specific operations impact the topological properties of spaces and analyzed the implications of these transformations. He also explored how these operations affect the topological structure of a space. By analyzing these operations, he provided insights into the preservation of important properties like openness, closedness, and connectedness under various transformations. Furthermore, he investigated the relationships between different types of operations and their impact on the resulting topology. He combined multiple operations to generate new topological properties and structures. His research shed light on the interplay between different operations and their effects on the overall structure of topological spaces. Following his work, to analyze the topological properties precisely, many researchers worked on operations on different open sets in topological space.

Asaad (2017) defined the concept of an operation on generalized open sets and studied its applications. Asaad and Ahmad (2017) introduced the notion of sg-γ-open sets by defining the operation on semi-generalized open sets in topological spaces. After that, Jayashree and Sivakamasundari (2018) introduced an operation κ on generalized-semi open sets and studied κ-open sets and some of its properties in topological spaces. Later, Asaad and Ameen (2019) introduced the notion of $g\alpha\gamma$-open sets by defining the operation γ on $g\alpha$-open sets and investigated separation axioms using $g\alpha\gamma$-open sets in topological spaces.

After the introduction of operation on topological spaces by Kasahara (1979), various authors recently have introduced operation approach concept in various other topological spaces. El-Sheikh and El-Sayed (2020) extended the concept of γ-operation in fuzzy soft ideal topological spaces. They defined γ-operation on the collection of fuzzy soft sets and analyzed its properties.

Asaad et al. (2020) put forth the notion of operation γ on supra topological space and introduced supra γ-open sets. The notions of μ_γ-g.closed sets on the subspace were investigated by them. They also introduced few new μ_γ-separation axioms and obtained the relationships between them. They presented some characterizations of the new functions via operator γ, some properties of $S_{(\gamma,\beta)}$ closed graph and strongly $S_{(\gamma,\beta)}$-closed graph.

Jamil (2021) studied several characterizations and properties of strong soft γ-pre-open sets. They investigated the relationships between strong soft γ-pre-open

sets and other types of soft open sets. They discussed the properties of the strong soft γ-pre-interior and closure of a subset. He also defined locally strong soft γ-pre-closed sets using strong soft γ-pre-closed sets and derived several results.

Roy and Noiri (2021) investigated the features of $\gamma\mu$ open sets by defining operation on generalized topological spaces.

The notion of soft bioperators $\tilde{\gamma}$ and $\tilde{\gamma}'$ were initiated by Asaad et al. (2021). They analyzed the properties of soft $(\tilde{\gamma}, \tilde{\gamma}')$-open sets and proved that every soft $(\tilde{\gamma}, \tilde{\gamma}')$-open set is soft open and the converse is true only when the soft topological space is soft $(\tilde{\gamma}, \tilde{\gamma}')$-regular. They also introduced the notions of soft $(\tilde{\gamma}, \tilde{\gamma}')$-$g$.closed sets and soft $(\tilde{\gamma}, \tilde{\gamma}')$-$T_{1/2}$ spaces and explored their fundamental properties.

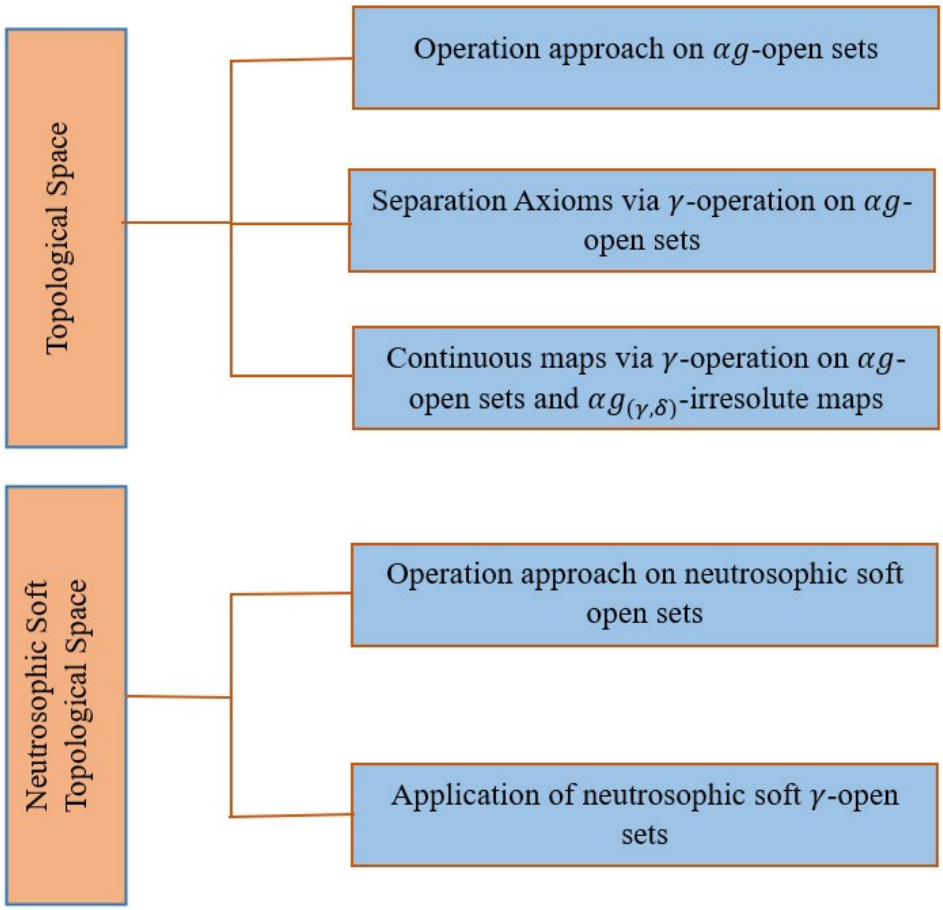
Das et al. (2022) explored the existence of operation approach on open sets in a generalized fuzzy topological space. They introduced the concept of fuzzy γ_μ-open set and studied some basic properties of it. They also obtained the result that the intersection of two fuzzy γ_μ-open sets need not be a fuzzy γ_μ-open set and proved that if the operation is regular, the intersection becomes a fuzzy γ_μ-open set. They also initiated the notions of fuzzy minimal γ_μ-open set and fuzzy γ_μ-locally finite space and established various results related to defined concepts.

Zadeh (1965) developed the fuzzy set ($\mathscr{FS}$) theory. It has developed as a very important tool for addressing problems with uncertainity. Molodtsov (1999) presented the soft set theory in 1991, which addresses uncertainty. Maji et al. (2003) established a theoretical approach to soft set theory and defined the operations like intersection (AND), union (OR) of two soft sets and justified some propositions on soft set operations. The notion of soft topological spaces, which are built over an initial universe with a predetermined set of attributes was developed by Shabir and Naz (2011).

Smarandache (1998, 2005) presented the novel idea of a neutrosophic set ($\mathscr{NS}$) which is a generalization of crisp sets, fuzzy sets, and intuitionistic fuzzy sets. Maji (2013), by integrating the idea of soft set and neutrosophic set, defined neutrosophic soft sets ($\mathscr{N}_\mathscr{S}\mathscr{S}$s) and provided an application of neutrosophic soft set in decision-making problems, which was later refined by Deli and Broumi (2015). Bera and Mahapatra (2017) constructed a topological structure on neutrosophic soft sets and studied its structural characterizations and discussed the concepts related to topological space.

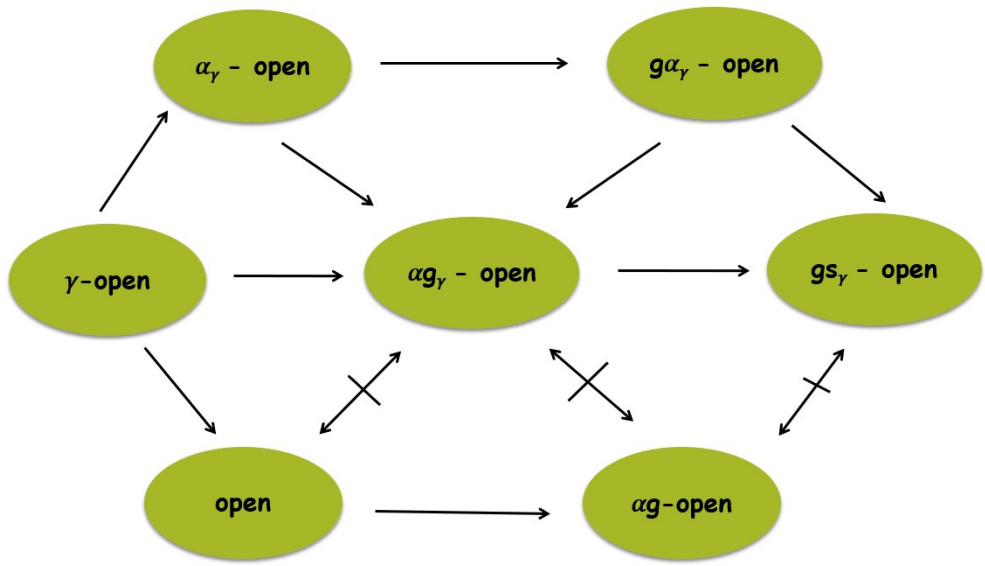
In our research work, a new class of sets named αg_γ-open sets is introduced by defining the operation γ on the collection αg-open sets. The fundamental properties of αg_γ-open sets and continuity concepts, separation axioms using αg_γ-open sets are analyzed. Then, the operation γ is extended to neutrosophic soft environment to bring forth new kind of soft set called neutrosophic soft γ-open set and its properties are examined in neutrosophic soft topological space. An algorithm is proposed to determine the optimal alternative in a decision-making problem using neutrosophic soft γ-open set and a python program is developed for the proposed algorithm.

The Outline of the research is presented here.

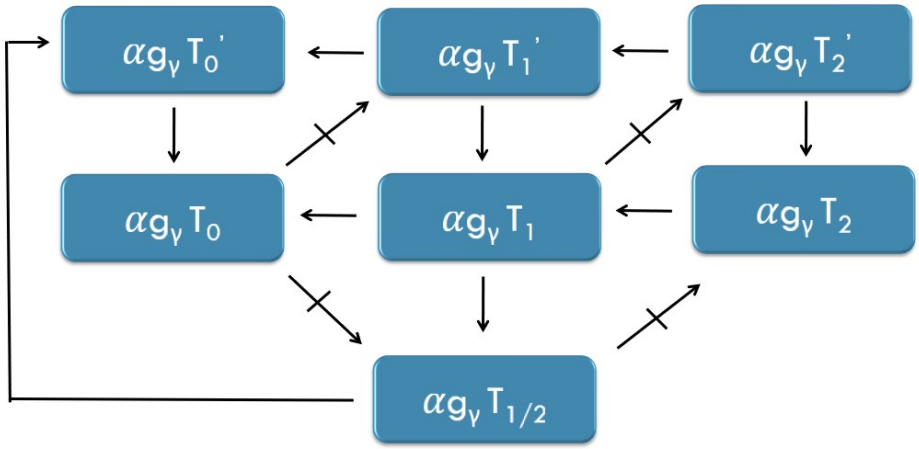

Topological Space
Operation approach on αg-open sets
Separation Axioms via γ-operation on αg-open sets
Continuous maps via γ-operation on αg-open sets and $\alpha g_{(\gamma,\delta)}$-irresolute maps
Neutrosophic Soft Topological Space
Operation approach on neutrosophic soft open sets
Application of neutrosophic soft γ-open sets

Preliminary definitions used in the investigation are arranged appropriately in Chapter 1. Also, Definitions of closed set, open set, generalized closed sets, operation approach on open sets and various generalized open sets, γ-separation axioms, γ-continuity, γ-open set in soft and fuzzy topology are grouped in this chapter.

In chapter 2, a new form of open set called αg_γ-open set is introduced. The fundamental characteristics of the αg_γ-open sets are systematically deduced with plenty of counter-examples. The dependency and independency relations between the newly established set and the previously existing sets are also depicted diagrammatically. Two different kinds of closure and interior operators are introduced using the proposed set and its properties are studied. Also the properties of αg_γ-kernel operator are derived. Finally, some theorems on αg_γ-generalized closed sets are studied.



Chapter 3 is dealt with the study of separation axioms using γ-operation on αg-sets, αg_γ-sets in topological spaces. The newly developed separation axioms comprise of $\alpha g_\gamma - T_i \ \ (i = 0,\ 1,\ 2,\ 1/2)$, $\alpha g_\gamma - T_i^{'} \ (i = 0,\ 1,\ 2)$, αg-γ-regular and αg-γ-normal spaces. Few characterization theorems of the defined separation axioms are investigated. Relationship between the defined axioms are derived and counter-examples are provided for clear understanding of the idea.



Different forms of continuity and homeomorphism concepts using γ-operation on αg-open sets in topological spaces are elucidated in Chapter 4. To comprehend the idea of continuity with respect to the γ-operation on αg-open sets, the notion of $\alpha g_{(\gamma,\delta)}$-irresolute maps is introduced. The relationship between the defined continuity concepts is derived. A result analogous to the Urysohn lemma is presented on αg-γ-Normal space. Some theorems are derived using $\alpha g_{(\gamma,\delta)}$-open and $\alpha g_{(\gamma,\delta)}$-closed maps.

Chapter 5 is dedicated to the study of the introduction of neutrosophic soft γ-open sets in neutrosophic soft topological spaces and the operation γ on neutrosophic soft open sets. Furthermore, using neutrosophic soft γ-closed and neutrosophic soft γ-open sets, the characteristics of closure and interior operators are examined. The idea of neutrosophic soft γ-open sets is then applied to solve a decision-making problem.

In Chapter 6, python programs are developed for the topological operators in neutrosophic soft topological space and for a decision making problem to make the computation process easier and faster.

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